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THE ORDER OF BATTLE IN THE ROMAN ARMY: EVIDENCE FROM MARCHING CAMPS

Summary. *The ratio of the length to width of a marching camp is called the* aspect ratio. *Ninety out of 93 camps had aspect ratios of whole numbers where the larger number was one digit greater than the smaller,* $(n/n + 1)$. *The commonest was 2/3, which is near to the* golden section, *a proportion of some mystic significance to the ancients. By using the* $n/(n + 1)$ *series the aspect ratio was kept between the 2/3 ratio and the square, another figure of mystic significance. The 2/3 ratio occurs in many aspects of Roman military organisation and was probably due to Pythagorean influences. The acreage of the camp within its ramparts approximates the number of* notional *cohorts encamped, that is in terms of multiples of a standard legionary cohort. In Scotland, Roman armies often operated in groups of eight notional cohorts and multiples thereof. The area-frequency distribution of camps in England indicated that a standard Roman army comprised* $32(8 \times 4)$ *notional cohorts. Such a force made a camp of 100* actus quadrati *area with a one* actus intervallum. *Roman texts suggest it was centred on a legion with typically, though not invariably, an order of battle of 12 legionary, 10 auxiliary infantry and 10 auxiliary cavalry (80* turmae) *notional cohorts. This grouping was both sub-divided and amalgamated into forces that were deployed in the field where they made their own appropriately sized camps. A 32 notional cohort force was intended to hold the Hadrianic* limes *in Britain, probably as a separate command from the legions and auxiliaries in reserve.*

INTRODUCTION

In a recent paper the author proposed the steps by which the Romans may have calculated the dimensions of their marching camps, once they knew the area required (Richardson 1997). The study considered 18 examples and suggested that camps were founded on regular rectangles whose sides, or axes, were defined by a limited number of whole number (integer) ratios. Because the long axis of a Roman land survey was the *decumanus*, a term that seemed to correspond with the *via decumana* of a camp, the terms *cardo* and *decumanus* were adopted for the camp's short and long axes. The ratio of *cardo* to *decumanus*, c/d, was known as the *aspect ratio* and the examples studied formed a series in which the second integer was always one more than the first, i.e., 2/3, 3/4, 4/5 and so on. Algebraically, this relationship can be written, $n/n + 1$ and was probably used to calculate the camp's length and breadth from certain suggested formulae.

A more recent paper suggested how the required camp area for an imperial army might have been calculated (Richardson 2000). The total strigified area was the sum of the space given to each unit; 1.5 *actus quadrati* (*a.q.*) to a legionary infantry cohort and 1/8th of this (0.1875 *a.q.*) to a *turma*. The strigified area was then multiplied by 1.333 (4/3) to give the area (A) within the *intervallum*. But because $1.5 \times 1.333 = 2$, doubling the number of infantry cohorts, having counted each *turma* as 1/8th, also gave the area A. The *intervallum* width was 1/8th of the square root of this area, the inner face of the rampart being reached by extending the axes by that amount. The rampart and berm would generally account for another 30 feet to the inner lip of the ditch. This sequence produced a camp whose area in statute acres within the ramparts roughly equalled the number of infantry cohort equivalents it contained, bearing in mind that eight *turmae* equalled one cohort. The actual correlation was one cohort to 0.981 acres. This observation validated the general reliability of comparing of camps by acreage. Furthermore, the paradigm permitted a simple computer spreadsheet to find the dimensions of a camp for any given force. Used iteratively, that is by changing the cohort and *turmae* numbers, the spreadsheet could simulate the dimensions of known camps.

This paper now addresses three matters arising; first, the probable rules of thumb used by camp prefects to calculate the area within the *intervallum*; second, the significance of the aspect ratios found in a larger sample of camps, and third, the possibility of gaining insights into the army's organisation from the relative frequency of variously sized camps.

THE AREA WITHIN THE *INTERVALLUM*

To determine the area within the *intervallum* it would have sufficed simply to remember the areas needed by each type of unit. These values could be found by multiplying the area of *strigae* of each unit by 1.33 and they are summarised in Table 1.

ASPECT RATIOS

Ninety-three camps were studied, mostly from England (Welfare and Swan 1995) but also with examples from Wales, Scotland (St Joseph 1969, 1973, 1977) and Israel (Schulten 1933). Camps with indistinct outlines were excluded. The dimensions were taken with a ruler from the published plans at the inner face of the rampart or from the ditches where outlines were known only by ditch crop marks. With awkward outlines, especially parallelograms and

TABLE 1
Areas within the camp required by different units: (*areas in actus quadrati*)

Unit	Numbers of men			*strigae*	Space allocation within the intervallum
	Infantry	Cavalry	(*turmae*)		
leg. coh.	480			1.5	2
coh. ped. quin.	480			1.5	2
coh. (600 strong)	600			1.875	1½
1st leg. coh.	800			2.5	3 1/3rd
coh. ped. mil.	800			2.5	3 1/3rd
coh. eq. quin.	480	120	(4)	2.25	3
coh. eq. mil.	800	240	(8)	4	5 1/3rd
ala quin		480	(16)	3	4
ala mil.		720	(24)	4.5	6

off-square figures, the rectangle that appeared to define the perimeter was drawn in pencil and then measured. Sometimes the average length of opposite sides was taken. The data were then converted into *actus* with a computer spreadsheet and the aspect ratio found by dividing the lengths of *cardo* and *decumanus* by the difference between them. Thus, where C and D were the actual measurements, the values for the ratio c/d were given by the formulae $c = C/(D - C)$ and $d = D/(D - C)$. Sometimes the first measurement did not quite give whole number values for c and d but very little manipulation of the dimensions corrected this; the discrepancies were, given the map scales, within reasonable observer error (within one per cent).

Ninety of the ratios conformed to the rule $n/n + 1$. The three exceptions were Cawthorn C, 1/3, the reduced camp at Uffington (camp 2) 3/5 and Sills Burn (South) 3/8. Over half (52%) fell within the range 1/1 to 5/6 and most of these were 2/3 (29%) and 4/5 (15%). Indeed, it is likely that all those above 9/10 would have been square (1/1) had their precise dimensions been known. Table 2 lists the findings and Table 3 shows the distribution of the aspect ratios.

The commonest ratio, 2/3, was, according to Hyginus (xxi), the preferred option, though he mentioned those that were 'more square'. The effect of the progression $n/(n + 1)$ was to move the 2/3 oblong, stepwise, towards a square, so where a camp outline could not be maintained 2/3 or square, the $n/n + 1$ system kept it in between the two.

The square and the 2/3 ratio appear to have been figures of some mystical significance to the ancients on account of the aesthetic qualities of their proportions. The 2:3 ratio is a close approximation to the *golden section*, a ratio based upon the notion of a line divided such that

TABLE 2
Aspect Ratios of Some Roman Camps

Ratio	(No.)	Camps
1/1	(13)	Grindon School, Grindon Hill, Coesike W1, Bowes Moor, Swine Hill 2, Brown Dykes, Walwick Fell, Troutbeck 3, Rey Cross, Featherwood E, Chew Green 1, Crackenthorpe, Cawthorn A.
1/2	(6)	Moss Side 1, Bootham Stray 1, Broomby Lane 2, Glenwhelt Leazes, Haltwhistle Burn, Milestone House.
1/3	(1)	Cawthorn C
2/3	(27)	Barrockside, Sunny Rigg 1, Crooks, Quatt, Farnley 3, Twice Brewed, Lees Hall, Kirkby Thore 2, Burlington 2, Seatsides 2, Burnhead, Bagraw S, Bagraw N, Chew Green 3, Dargues, St Harmon, Fell End, Horstead, Walford, Ancaster, E Learmouth, Norton, Swindon, Silloans Featherwood W, Bellshiel, Uffington 1.
3/4	(3)	Burlington 1, Greensforge 5, Willowford.
3/5	(1)	Uffington 2
3/8	(1)	Sills Burn S
4/5	(14)	Golden Fleece, Sunny Rigg 2, Chapel Rigg, Cawfields, Dun, Ardoch 2, Greenlee Lough, Knowe Fm, Gleadthorpe, Masada B, Markham Cottage. 2 Kirkby Thore 1, Bromfield, Markham Cottage 1.
5/6	(9)	Sills Burn N, Masada A, Cawthorn D, Masada F1, Broomby Lane 1, Cawthorn B, Birdhope 1, Stracathro, Brampton Bryan
6/7	(4)	Malham, W. Woodburn, Troutbeck 2, Kirkby Thore 3.
7/8	(4)	Coesike E, Nowtler Hill 1, Langwathby, Farnsfield.
8/9	(1)	Caerau
9/10	(1)	Esgairperffed
10/11	(5)	Seatsides 1, Chew Green 4, Birdhope 2, Moss Side 2, Troutbeck 1
11/12	(0)	
12/13	(0)	
13/14	(1)	Wath
14/15	(0)	
15/16	(1)	Swine Hill 1
20/21	(1)	N Yardhope
Total	93	

TABLE 3
Frequency of Aspect Ratios in 93 Roman Camps

Ratio	%	Examples	Ratio	%
1:1	14		9:10	1
1:2	6		10:11	5
1:3	1	Cawthorn C	11:12	0
2:3	29		12:13	
3:4	3		13:14	1
3:5	1	Uffington 2	14:15	
3:8	1	Sills Burn S	15:16	1
4:5	15		16:17	
5:6	10		17:18	
6:7	4		18:19	
7:8	4		19:20	
8:9	1		20:21	
			Total	100

the ratio of the smaller part to the large is the same as the larger part to the whole (Anon 1986). But it cannot be defined in whole numbers with exactitude. For example, whereas 2/3 is 0.666 recurring, the golden section is 0.618 with a never-ending sequence of decimal places, like the value for π. So if the square be regarded as the starting point, the $n/n + 1$ series (1/1, 1/2) takes the square towards the golden section, but no further than 2/3. However, the first two ratios start another series that may be continued by adding the last two numbers above and below the line to give the next ratio. For example, 1/1, 1/2, give 2/3, i.e., $1 + 1 = 2$ and $1 + 2 = 3$, then 3/5, 5/8, 8/13 and so on. This progression is known as the Fibonacci series from the Renaissance scholar who discovered it and if the process is continued, the golden section (0.618) is reached at 34/55 but thereafter is never exceeded. The only ratio in the Fibonacci series beyond 2/3 found with the camps, is 3/5 (Uffington 2). This raises the possibility that long before Fibonacci, the ancients were aware of how this series led to the golden section. The two camps that stand outside either series are Cawthorn C (1/3) and Sills Burn South (3/8). These are simple enough ratios, and no doubt just as practical as the others so their very rarity may imply a policy of preferring ratios of some mystic significance.

This idea receives some support from the fact that the 2/3 ratio, the simplest and most practical approximation to the *golden section*, is seen in the subdivision of the area within the camp. The effect of defining the *intervallum* width as 1/8th square root of the area it encloses (A) is to make the area of the *intervallum* about 1/3 of the camp area leaving 2/3 for the rest (Richardson 2000). In the Polybian camp, the cavalry had virtually 1/3 of the area within the *intervallum* and the infantry 2/3 and in the Hyginian model, each nominal cohort had 3/2, i.e. 1.5 *actus quadrati* (*a.q.*) of *strigae* (Richardson 2000).

NUMERICAL RATIOS IN ARMY ORGANISATION

In the consular army of the republic the infantry comprised 120 maniples of different numbers but with a mean of 140 men and the cavalry of 80 *turmae* of 30. Thus, the ratio of *turmae* to maniples was 80:120, or 2/3. The ratio of legionaries to *socii*, was 80:120, or 2/3. There were equal numbers of legionary and allied infantry but the ratio of *turmae* was 20 legionary to 60 *socii*, 2/3. See Table 4. The same ratio also defines quingenary to milliary *alae* (in *turmae*) of the imperial army, 16 to 24. Moreover, the first imperial legionary cohort

TABLE 4

Composition of Forces of the Consular Army described by Polybius: Numbers of men with units (maniples or *turmae*) in brackets

		Legionary			Allied			Totals
Infantry		4,200	(30)		4,200	(30)		
		4,200	(30)		4,200	(30)		
	Sub total			(60)			(60)	(120)
Cavalry		300	(10)		900	(30)		
		300	(10)		900	(30)		
	Sub total			(20)			(60)	(80)
Total Units				(80)			(120)	(200)

comprised 800 men in five doubled centuries (80 to 160) and so having 320 more men, it was a standard cohort made bigger by 2/3; $[480 + (480 \times 2/3) = 800]$.

CAMP AREA AND ARMY ORGANISATION

Because the camp area reflected closely the composition of the army that made it, there may be clues as to the composition of armies to be gleaned from the relative frequency areas of camps of given area. The fact that a standard infantry cohort occupied the same space as eight *turmae* means that we have a common denominator in area terms for both infantry and cavalry units. For example, the *ala quingenaria*, comprising 16 *turmae* was equivalent to two infantry cohorts. The author initially used the term 'nominal cohort' to mean any unit that took up the same space as a legionary cohort so an *ala quingenaria* equalled two nominal cohorts (Richardson 2000). But because the term *nominal* could be misleading in the context of the legion, it will be replaced by the term *notional*.

The notional cohort thus provides a sort of common currency for the various infantry and cavalry units and the Romans appear to have used it, or something analogous, for the conceptual structure of the army. The notional cohort, however it was composed, accounted for two *a.q.* within the *intervallum* and contributed 0.5 *a.q.* to the non-strigified area into which the roads and other facilities were fitted. The notional cohort values for units of the imperial army can be calculated by taking their area of *strigae* from the data of Hyginus and dividing by the area of *strigae* of the legionary cohort, 1.5. The values are summarised in Table 5 for which the troop numbers are taken from Breeze and Dobson (2000, 161).

TABLE 5

Notional Cohorts: (*area of strigae divided by 1.5*)

Unit	*strigae (a.q.)*	Notional value
Legionary cohort	1.5	1
1st Leg. cohort	2.5	1.666
coh. ped. quingenaria.	1.5	1
600 man cohort	1.875	1.25
coh. ped. mil.	2.5	1.666
coh. equi. Quin.	2.25	1.5
coh. equi. Mil.	4	2.666
Ala quingenaria	3	2
Ala milliaria	4.5	3

According to this scheme the first legionary cohort counts as 1.66 notional cohorts, so the whole legionary infantry comprises 10.66 notional cohorts. Since the legionary cavalry numbered 120, they would occupy 0.75 *a.q.* of *strigae* and count as 0.5 notional cohorts. The total for the whole legion would be 11.16. This seems an awkward number and for simplicity and at no great cost in accuracy, given its supernumerary staff, a legion probably rated as 12 notional cohorts. Auxiliary infantry cohorts probably counted as legionary cohorts even when 500 strong for the extra 20 men would take up negligible extra space. If each infantryman had the usual 45 sq. feet (Richardson 2000) cohorts of 600 would have 1.875 *s.q.* of *strigae*, $[(45 \times 600)/14400]$ and rate as 1.25 notional cohorts.

Bearing in mind the virtually direct correlation of notional cohorts with a camp's acreage we may now enquire into the possible composition of the armies that made certain camps.

CAMP ACREAGE IN SCOTLAND

St Joseph (1969, 1973, 1977) has listed the acreages of many camps in Scotland. Here it seems the Roman army often operated in multiples of eight notional cohorts since many groups of camps differ in area by roughly 8, 16 and 24 acres. Moreover, there are several camps differing by about $32(4 \times 8)$ acres giving two groups of 32 and 63 (*circa* 64) acres. Rae Dykes is about $96(3 \times 32)$ acres. Then there are those which are multiples of 32, the $128(4 \times 32)$ acres series and the $160 (5 \times 32)$ acre camps. Durno is $128 + 16 = 144$ acres. Table 6 lists these camps and though the examples cited might date from different periods, the deployment of eight notional cohorts and multiples thereof was probably routine over a long period.

The army group from which these putative eight notional cohorts were drawn was almost certainly centred on a legion which, according to Vegetius, was attended by about 4,000 auxiliary infantry and 2,000 cavalry (Milner 1996, 65). These numbers suggest an auxiliary component of eight infantry cohorts and four *alae* of 16 *turmae* and each equal to two notional

TABLE 6
Approximate Acreages of Some Camps in Scotland: (*after St Joseph 1969, 1973, 1977*)

Area (differences in bold type)	Examples
24	Dornoch, Mentieth, Dalingross
8	
32	Dunblane, Ardoch
8	
40	Stracathro, Castledykes
24	
64	19 camps (St Joseph 1973, 230)
24	
96	Rae Dykes (93)
16	
112	Ythan Wells, Dunning
16	
128	Ardoch, Grassy Walls, Cardean
16	
144	Durno
16	
160	Newstead

cohorts. Allowing 12 notional cohorts to the legion, this army group would comprise 12 + 8 + 8 (total 28) notional cohorts and should have had a camp of about 28 acres within the ramparts. But Vegetius wrote long after the army was at its prime so his testimony may not be accurate in detail.

The Hyginian army, comprising three legions with their auxiliaries, seems to have been similarly structured. The author has suggested that with 11 notional cohorts to the legion, the infantry of this army had 95 *a.q.* and the cavalry 70 *a.q.* of *strigae* (Richardson 2000). Dividing these areas by 1.5 tells us the force comprised 63.33 notional cohorts of infantry and 46.66 of cavalry, a total of 110. Now realising that a legion is better regarded as 12 notional cohorts, the estimate can be revised. The infantry become 63.33 + 3 = 66.33 notional cohorts, of which 36 are legionary, leaving 30.33, say 30, auxiliary. Therefore with each legion went ten notional auxiliary infantry cohorts and 46.66 notional cavalry cohorts, or about 15.5 each legion. In approximate terms this order of battle could be written, 12 + 10 + 15, total 37, or five over the theoretical 32. This difference, however, could be due to the presence of the emperor and praetorian troops in this particular camp. In short, the information from both Vegetius and Hyginus is not inconsistent with the idea that a legion and its compliment of auxiliaries would have had a camp of about 30 to 32 acres.

SIZE AND FREQUENCY OF CAMPS IN ENGLAND

In Britain, camps of 30 to 32 acres are not common, suggesting that the putative army group based on a legion was usually broken down into smaller units or combined to make larger forces. This idea receives support from the monograph on camps in England by Welfare and Swan (1995, 11) which contains a bar chart of the frequency of camps in area categories. The camp areas were within the ramparts or ditch crop marks, according to the evidence from the site (Welfare, pers. comm.). Now, most significantly, the authors observed that as camp size increased, the frequency decreased and they recognised the relationship was consistent with the notion of a large force being continually halved to produce twice the number of smaller components. This phenomenon is analogous to the decay curve of a radioisotope, or the half-life of a drug in the blood stream. It means that a graph plot of the numbers of camps in each area category (frequency) against the logarithm of the area should give a straight line, and this, in turn, should permit the data to be subjected to the statistical process known as regression analysis. The graph line's *intercept*, that is the point where it cuts the logarithm area axis, would be the camp area of that force that was subdivided to give all the smaller camps; that is the camp analogous to the dose of drug before its degradation in the blood.

The results of this exercise are shown in Table 7. For convenience, the upper limit of each area category rather than its mid point, was used. The correlation was highly significant, 1000 to 1 against a chance result (coefficient -0.80, $p < 0.001$). The intercept was logarithm 2, which is exactly 100, so the camp that was the parent of all the others was 100 *a.q.* within the ramparts which is 32 acres. The synthesis model shows that this camp had an *intervallum* of one *actus*, enclosing an area of 64 *a.q.*, within which were 48 *a.q.* of *strigae* for 32 notional cohorts. The simplicity of the configuration (in *actus*) is striking, 8×8 within the *intervallum* and 10×10 at the inner face of the rampart. Table 8, summarises this spreadsheet. Three camps in England, East Learmouth (33.6 acres), Greensforge 3 (32.7 acres) and Norton 1 (32 acres) appear to be close examples (acreages from Welfare and Swan 1995).

TABLE 7
Frequency of Camp Areas in England: (*after Welfare and Swan 1995, a.q. = actus quadrati*)

No of camps	hectares	*a.q.*	Log. *a.q.*
37	1	7.7	0.888
20	2	15.44	1.189
5	3	23.17	1.365
9	4	30.89	1.490
3	5	38.61	1.587
2	6	46.33	1.666
2	7	54.05	1.733
3	8	61.77	1.791
5	9	69.50	1.842
5	10	77.22	1.888
2	12	92.66	1.967
1	13	100.38	2.002
3	14	108.10	2.034
6	16	123.55	2.092
2	17	131.27	2.118
2	19	146.71	2.166
1	24	185.32	2.268
Correlation coefficient	−0.80		
Intercept	2.0		

TABLE 8
Spreadsheet of a 32 Notional Cohort Camp:

In-put $(n = 32)$ and out puts		Formulae		Values
Notional cohorts	(n)			32
Area of *strigae*	(S)	$n \times 1.5$		48
Area within *intervallum*	(A)	$n \times 2$		64
Width of *intervallum*	(I)	1/8th $\sqrt{A}$		1
Area at inner face of rampart	(R)	$(\sqrt{A} + 2I)$ squared	100	(32.0 acres)

COMPOSITION OF THE 32 NOTIONAL COHORT ARMY

The 32 notional cohort army almost certainly was grouped around a legion which would account for 12 notional cohorts, leaving 20 for the auxiliaries. In theory, these could have been partitioned in any ratio from one infantry to 19 cavalry, or vice versa, but in practice the division was probably somewhere about the middle, as Hyginus and Vegetius imply. There are, however, other clues that narrow down the probabilities.

Table 9 shows the numbers of infantrymen per cavalryman in certain situations. In the Polybian camp there were seven infantrymen per cavalryman, but in the imperial army the proportion of cavalry relative to infantry was roughly doubled, although it is hard to be certain because Hyginus does not always make clear which of the tribal units were infantry and which were cavalry. In the mixed units of the empire there were 3.75 infantry per cavalryman in the milliary cohort and 3.125 in the quingenary.

Bearing in mind these findings, the infantry-cavalry permutations that could make up 32 notional cohorts, presuming 120 legionary cavalry to the legion, were found to be as shown

TABLE 9
Number of Infantrymen per Cavalryman in Typical Roman Forces

Force	infantry	cavalry	Infantry/cavalry	Reference
Polybian	16,800	2,400	7	
Cohors equitata mil.	800	256	3.12	Breeze & Dobson (1976)
Cohors equitata quin.	480	128	3.75	ditto

TABLE 10
Some Options for the Partition of Auxiliaries Brigaded with One Legion: (*Totals of men include 5,600 legionaries with 120 cavalry*)

Options	1	2	3	4	5
Auxiliary infantry cohorts	10	9	8	7	5
Notional auxiliary cavalry cohorts	10	11	12	13	15
Auxiliary *turmae* (cohorts × 8)	80	88	96	104	120
Infantrymen	9,920	9,440	8,960	8,480	7,520
Cavalrymen	2,520	2,760	3,000	3,240	3,720
Total men	12,440	12,200	11,960	11,720	11,240
No *turmae* to auxiliary cohorts	8.0	9.8	12.0	14.9	24
No *turmae* to all cohorts	2.4	2.7	3.0	3.4	4.1
Infantrymen per cavalryman	3.9	3.4	3.0	2.6	2.0

in Table 10. Option three, eight auxiliary infantry notional cohorts to 12 cavalry (96 *turmae*) gave three infantrymen per cavalryman. Ten notional cohorts of each gave a ratio of 3.9 and the intermediate positions of nine to 11, gave a ratio of 3.4, very similar to those in the mixed units. Indeed, it would appear that the mixed units were intended to be a microcosm of the whole army. Option three has the merit of having whole numbers, and the 96 *turmae* would account for six *alae quingenariae*. The other similarly plausible model is option one, which gives five *alae quingenariae*. In both cases the ratio of *turmae* per infantry cohort is one of whole numbers which would have facilitated combining infantry and cavalry in small numbers.

HISTORICAL EVIDENCE

It is now necessary to see whether the historical record contains evidence to support any of these suggested orders of battle. Tacitus and Josephus provide snap shot evidence of army composition on certain occasions. In Tacitus' account of the Mons Graupius campaign, Agricola's auxiliary component was 8,000 infantry (16 cohorts of 500) and 5,000 cavalry. If we take his cavalry figure to mean ten *alae* each of 16 *turmae*, they count as 160 *turmae*, or (160/8) notional cohorts, and if this force were attached to two legions, the order of the whole army would have been $24 + 16 + 20$ (60). The order with one legion would have been half this, $12 + 8 + 10$ (30), closer to the suggested model than the $12 + 10 + 15$ (37) detectable in Hyginus.

This observation can be supplemented from other passages. Table 11 shows details from 12 armies, including that of Agricola mentioned above, whose compositions can be reasonably reconstructed from the texts. Seven contained legions and five apparently did not. Legions were interpreted as 12 notional cohorts and the thousands of infantry at 500 per cohort, unless otherwise stated. Where Josephus (3,71) mentions ten 'double' cohorts they were

counted as 20 notional cohorts. A troop of 500 cavalry was regarded as an *ala* of 16 *turmae*, strictly speaking 480 men at 30 per *turma* or 512 with 32 men per *turma*, but both equalling two notional cohorts. Josephus quite specifically states that each of thirteen 600 strong auxiliary cohorts were brigaded with four *turmae*, and that ten milliary infantry cohorts were brigaded with six cavalry troops (96 *turmae*). The 13,000 men that Mucianus brigaded with the 6th Legion to aid Vespasian (Tacitus, Histories, 2,83) were taken as divided similarly to Agricola's Mons Graupius army.

Table 11 gives the total notional cohorts for each army and the mean order of battle derived from all 12. These forces totalled about 170,000 men and their mean order per legion was $-12 + 11 + 8 \, (31)$ *notional cohorts*.

Table 12 summarises the order of battle for the seven armies where legions were brigaded with auxiliaries. The mean order was $12 + 7.9 + 7.5$, say $12 + 8 + 8 \, (28)$ or four notional cohorts short of the putative model. The force that Titus and Vespasian assembled in Palestine (Josephus 2, 71) comprising three legions each with an order $12 + 12 + 6 \, (30$ was the nearest to the model. In general, the historic evidence of battle order is not inconsistent with the notion that, in theory, 20 notional auxiliary cohorts, about half of them cavalry, should have attended each legion.

TABLE 11
Order of Battle in Notional Cohorts in Twelve Armies: (*from Tacitus and Josephus*)

Reference	legionary cohorts	auxiliary cohorts infantry	cavalry (*turmae*)	totals	
Jos. 3, 71: *3 legions*	36			36	
10 double cohorts,		20		20	
13 cohorts (600) each with 4		16.25	6.5 (52)	22.75	
turmae, 6 cavalry alae.			12 (96)	12	
					90.75
Tac. Hist. 2, 25: *1 legion*	12			12	
2 cohorts, 500 horse		2	2 (16)	4	
					16
Tac. Hist. 2, 25: *1 legion*	12			12	
4 cohorts, 500 horse		4	2 (16)		6
					18
Jos. 2, 498: *1 legion + 2000 men*	16			16	
6 cohorts, 4 alae		6	8 (64)	14	
					30
Tac. Agricola, *35: 2 legions*	24			24	
8000 infantry, 5000 horse.		16	20 (160)	36	
					60
Tac. Hist. 2, 83: *I legion*	12			12	
13,000 men		16	20 (160)	36	
					48
Tac. Hist. 2, 89: *4 legions*	48			48	
34 cohorts, 12 alae		34	24 (192)	58	
					106
Tac. Hist. 3, 58: 6 coh., 500 horse	—	6	2 (16)		**8**
Jos. 3, 293: 1000 foot, 500 horse	—	2	2 (16)		**4**
Jos. 3, 293: 2000 foot, 1000 horse	—	4	4 (32)		**8**
Jos, 3, 321: 3000 foot, 600 horse	—	6	(20)		**2.5**
Jos. 3, 46: 6000 foot, 1000 horse		12	4 (32)		**16**
Totals	**160**	**144.3**	**109**		**413.3**
Mean notional cohorts per legion	12	10.8	8.2		31

TABLE 12
Notional Cohorts per Legionary Group in Seven Armies: (*after Tacitus and Josephus*)

Reference	Legionary	Auxiliary		Totals
		infantry	cavalry	
Tacitus, Hist. 2, 83	12	16.0	20	48.0
Josephus, 3, 71	12	12.0	6.2	30.2
Tacitus, Agricola, 35	12	8.0	10	30.0
Tacitus, Hist. 2, 89	12	8.5	6.0	26.5
Josephus, 2, 498	12	4.5	6.0	22.5
Tacitus, Hist. 2, 25	12	4.0	2.0	18.0
Tacitus, Hist. 2, 25	12	2.0	2.0	16.0
Means	12	7.9	7.5	27.4

THE ARMY IN BRITAIN

A further insight into the order of battle order might come from the evidence of garrison strengths, though doubtless these would have changed over time. In Hadrianic Britain, three legions and numerous auxiliary cohorts were based behind a frontier held entirely by auxiliaries. Did these frontier forces brigade with the legions or did they constitute a separate command? In the first case, the putative model would predict that the auxiliary garrison should comprise $3 \times 20 = 60$ notional cohorts. In the second case the model would predict multiples of 20 above 60, i.e., 80, 100, 120, and so on.

Table 13 lists the units believed to be deployed initially on the Wall and the Solway coast. The identities of the units are taken from Breeze and Dobson (2000, 256–276) with their notional cohort values derived from Table 6. They total 31.8 notional cohorts, say 32, presuming a quingenary infantry unit at Newcastle and excluding Drumburgh whose status is uncertain and Burrow Walls which is believed to be later than Hadrian. Half the force (16 notional cohorts) comprised non-mixed units, nine infantry and seven cavalry, with the other half deployed in the mixed units. In the whole force there were 12.5 notional cohorts of cavalry (10 *turmae*, or 3000 men) and 19.33 of infantry. Seven of the notional infantry cohorts were deployed as purely infantry units in the central Wall area between Housesteads and Castlesteads with the two other (quingenary) infantry cohorts at Newcastle and Beckfoot. Overall, there were virtually three infantrymen per cavalryman. The total of 31.8 (32) notional cohorts is surely not accidental and it would seem that this frontier force, though entirely auxiliary, was analogous to a legionary army group whose battle order might be written $12 + 7.5 + 12.5$ (32).

Breeze and Dobson (2000, 163) set this frontier force within the context of a British auxiliary garrison that, on their figures, totalled 59 auxiliary units. If 59, why not a round 60? They state that only two *cohortes milliariae peditatae* are attested, listing them as *I Dacorum* at Bewcastle and *I Tungorum* at Birdoswald, but they also suggest that another, unidentified, was at Housesteads under Hadrian (Breeze and Dobson 2000, 258). If this unidentified unit were included in the garrison figures, the auxiliary garrison totalled 60 units and was partitioned between the frontier and the reserve as set out in Table 14, almost a third (19) to the frontier and two-thirds (41) to the reserve.

In notional cohort terms the British garrison comprised 100 (actually 99.33) notional cohorts and was thus met the third predicted option stated above. As with the frontier force the infantry — cavalry split was half-and-half. One quarter of the cavalry (100 *turmae*) were

TABLE 13
Notional Cohorts Deployed on the British Hadrianic Frontier:

Site	Unit	Infantry (men)	Cavalry (*turmae*)	Notional cohorts infantry	cavalry	totals
Infantry only						
Newcastle	c.q.p.	480		1		
Housesteads	c.m.p.	800		1.666		
Gt. Chesters	c.q.p.	480		1		
Carvoran	"	480		1		
Birdoswald	c.m.p.	800		1.666		
Bewcastle	"	800		1.666		
Beckfoot	c.q.p.	480		1		
Sub totals		4,320		9		9
Cavalry only						
Benwell	a.q.		480 (16)		2	
Chesters	a.q.		480 (16)		2	
Stanxix	a.m.		720 (24)		3	
Sub totals			1,680 (56)		7	7
Mixed units						
Wallsend	c.q.e.	480	120 (4)	1	0.5	
Rudchester	"	480	120 (4)	1	0.5	
Halton Chesters	"	480	120 (4)	1	0.5	
Carrawburgh	c.q.e.	480	120 (4)	1	0.5	
Castlesteads	"	480	120 (4)	1	0.5	
Burgh by Sands	"	480	120 (4)	1	0.5	
Bowness	c.m.e.	800	240 (8)	1.666	1	
Maryport	"	800	240 (8)	1.666	1	
Moresby	c.q.e.	480	124 (4)	1	0.5	
Sub totals		4,960	1,320 (44)	10.33	5.5	15.83
Grand totals		9,280	3,000 (100)	19.33	12.5	**31.8**

Abbreviation key: c = cohors, e = equitata,
p = peditata, q = quingenaria, m = milliaria

TABLE 14
Allocation of Auxiliary Units between the British Frontier and the Reserve

unit	frontier	reserve	totals
coh ped. quin	4	8	12
coh ped. mil	3	0	3
coh. eq. quin.	7	17	24
coh. eq. mil.	2	3	5
ala quin.	2	13	15
ala mil.	1	0	1
totals	19	41	60

deployed on the frontier and three quarters (300 *turmae*) were held in reserve. It would seem that even in the pragmatic task of defending Britain the Roman forces were organised and distributed according to notions of numerical proportion. See Table 15.

If the whole auxiliary garrison be regarded as brigaded with the legions, the order of battle of the army in Britain was $36 + 50 + 50$ (136), or per legion $12 + 16.66 + 16.66$ (45.33). This does not accord with the 32 notional cohort model. On the other hand, if the frontier

TABLE 15
The Auxiliary Garrison of Britain

units	number	actual numbers infantrymen	*turmae*	infantry	notional cohorts cavalry	totals
coh ped. quin	12	5,760	—	12	—	12
coh ped. mil	3	2,400	—	5	—	5
coh. eq. quin.	24	11,520	96	24	12	36
coh. eq. mil.	5	4,000	40	8.33	5	13.33
ala quin.	15	—	240	—	30	30
ala mil.	1	—	24	—	3	3
totals	**60**	**23,680**	**400**	**49.33**	**50**	**99.33**

garrison were an independent force, the remaining auxiliaries would comprise $50 - 19.33 = 30.66$ notional cohorts of infantry and $50 - 12.5 = 37.5$ of cavalry. This would give an order of battle of per legion of $12 + 10 + 12.5 = 34.5$, much closer to the putative ideal and this might indicate that the frontier force constituted a separate command.

ORDER OF BATTLE FROM THE POOLED DATA

If we pool the data for order of battle derived from the snapshots afforded by the historians with evidence from Britain, having regarded 12 notional infantry cohorts on the frontier as standing in for a legion, the average is $12 + 8.4 + 10.8$ (31.2). See Table 16. It is reasonable to conclude from the available evidence that the ideal theoretical auxiliary accompaniment of a legion was 20 notional cohorts, about half of which were cavalry.

DISCUSSION

The findings reported here suggest that the organisation of the Roman army and the plan of its camps were inter-dependent exercises; that is, the army was organised so as to make the laying out of camps straightforward. The camp's geometry and the internal distribution of troops seem to reflect an underlying idea of sacred mathematical harmony not readily appreciated by the modern mind. The military planners responsible for these matters were undoubtedly familiar with Greek mathematics and the quasi-religious reverence for certain numbers and ratios derived from the Pythagoreans. At first sight, it might seem that these numerical scruples fly could in the face of practicalities, but if the siting of the camp and the hour of battle could be influenced by the pathology of the sacrificial chickens, it is not surprising that divine numerical proportions were similarly respected. To get matters right

TABLE 16
Summary of evidence of the typical order of battle in notional cohorts

sources	legions	auxiliary infantry	auxiliary cavalry	totals
Historic (Table 12)	12	7.9	7.5	27.4
British *limes* (Table 13)	12*	7.33	12.5	31.8
British reserve	12	10.1	12.5	34.6
means	**12**	**8.4**	**10.8**	**31.2**

* Ten notional cohorts of infantry on the Wall are regarded as substituting for a legion.

numerically was to invite the approval of the gods; and because the mathematical paradigms led to efficiency, the gods did approve, rewarding the army with success in a self-fulfilling circular argument.

An analysis of the areas and frequencies of camps in Britain pointed to a basic army group of 32 notional cohorts whose camp had a one *actus intervallum* and covered 100 *a.q.* (32 acres) at the ramparts. Typically, but not necessarily invariably, it would comprise one legion, ten auxiliary infantry cohorts and 80 auxiliary *turmae* with a ratio of infantrymen to cavalrymen very similar to those seen in the mixed units. The evidence derived from Tacitus and Josephus suggests that often slightly fewer auxiliaries accompanied the legion, but with a greater variability in their make up. These historians may have inadvertently revealed some cavalry shortages, for emergencies must often have lead to variations.

This one legion battle group could be divided into four sub-groups of eight, or eight sub-groups of four, or several other even-numbered permutations of notional cohorts, and the range of known camp areas suggests that almost every possible combination was deployed. Likewise, forces comprising multiples of 32 notional cohorts would make camps of roughly 64, 96, 128 and 160 acres, of which there are many examples in Scotland. Indeed, north of the Wall there are camps that appear to have been the work of forces greater than the whole British garrison, which would have occupied a camp of about 136 acres. Thus these camps probably included the whole garrison plus drafts from the continent, or were made to take enormous numbers of prisoners and booty.

It is possible that the Roman military planners used the convention adopted in this paper of writing the order of battle as three numbers of notional cohorts whose sum, when doubled, gave the area needed within the *intervallum* for their camp. But the lay out scheme was probably broken down into simple operating orders for the men, and at a practical level the camp prefect probably used a rule of thumb like that shown in Table 1, simply memorising that each particular type of unit required a certain number of *a.q.* within the *intervallum*.

Through these military works we glimpse a lost vision of holiness, namely that depicted in the patterns of certain numbers and ratios. Ideas of sacred numerology continued to fascinate religious thinkers down to the time of the Manichaeans in the 3rd and 4th centuries (Grant 1974, 256) and it may be that instructions for the men were first couched in the form of religious, or at least superstitious, incantations. With the rise of Christianity these ancient rules may have been piously avoided and eventually lost. Certainly Hyginus and Vegetius were aware that the crucial methodology had gone and were unable to retrieve it.

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